The 10th Young Scholar Symposium of the East Asia Section of IPIA

Day 1

The plane wave method

Mikko Salo (University of Jyväskylä)

Abstract:
The inverse problem of Calderón, in its geometric formulation, asks if a Riemannian metric in a domain is determined up to isometry by boundary measurements of harmonic functions. Physically this corresponds to determining a matrix electrical conductivity function from voltage and current measurements on the boundary. This problem is open in general.

In this talk we will discuss the hyperbolic analogue of the Calderón problem, involving the (Lorentzian) wave equation in various settings. For time-independent coefficients this boils down to the classical Gel'fand or inverse boundary spectral problem, and there is a large literature since the 1980s based on the Boundary Control and geometrical optics methods.

Recently, a new method has been introduced for hyperbolic inverse problems. The method is based on (distorted) plane waves, and it involves Carleman estimates in the spirit of the Bukhgeim-Klibanov method. This method applies to certain formally determined inverse problems also with time-dependent coefficients. In this talk I will attempt to explain the basic ideas behind the plane wave method and the results obtained in this way.

This talk is based on joint work with L. Oksanen (Helsinki) and Rakesh (Delaware).

Single boundary measurement uniquely determines the spatially heterogeneous order of certain classes in time-fractional diffusion

Jiho Hong (The Chinese University of Hong Kong)

Abstract:
In this talk, we describe and compare our several uniqueness results for an inverse problem in an anomalous diffusion process. The inverse problem is to recover the heterogeneous media modeled by the spatially varying order of the time-fractional subdiffusion from a single boundary measurement. Monotonicity assumptions on the class of candidate order functions played a key role for the uniqueness under one-point data in 1D [H.–Jin–Kian, SIAM J. Math. Anal. 57(2)], 2D and 3D [H.–Jin–Kian, SIAM J. Math. Anal. 58(1)]. In contrast, we prove the uniqueness without monotonicity and with less regularity assumptions under full-boundary data in 2D, 3D and higher dimensions [H.–Jin–Kian, arXiv:2602.23037]. The proof utilizes various techniques from complex, asymptotic, and geometric analysis. This is a collaborative work with Bangti Jin and Yavar Kian.

Iterative Direct Sampling Method for Inverse Boundary Value Problems

Fengru Wang (The Chinese University of Hong Kong)

Abstract:
Inverse boundary problems aim to recover hidden internal properties from external measurements, but are challenging due to limited and noisy data. This talk introduces the Iterative Direct Sampling Method (IDSM), which enhances the classic Direct Sampling Method with an iterative refinement process. This approach improves reconstruction accuracy and stability significantly, even with partial data or strong noise. We will outline the key ideas behind IDSM and demonstrate its application to both static and dynamic imaging problems, showing it to be a robust and practical tool for scientific and engineering applications.

On the inverse inclusion problem for convex polyhedra

Chun-Hsiang Tsou (National Central University, Taiwan)

Abstract:
The inverse inclusion problem consists to consider the following boundary value problem.

$$\begin{cases} \operatorname{div}((1+(k-1)\chi_D)\nabla u) = 0 & \text{in} \quad \Omega, \\ u = f & \text{on} \quad \partial\Omega. \end{cases}$$

Here, Ω is a bounded Lipschitz domain in $\mathbb{R}^n$ and $D \Subset \Omega$ is an unknown region with its indicator function χ_D. The constant $k > 0$ represents the conductivity of the inclusion D and f is a given function on $\partial\Omega$. The main issue of the inverse inclusion problem is to determine the unknown region D from the boundary measurements $(f, \partial_\nu u)$.

This problem is classical in the field of inverse problems and known as the Calderón problem. Many uniqueness and stability results are well known for this problem through the Dirichlet-to-Neumann map $\Lambda_D : f \mapsto \partial_\nu u|_{\partial\Omega}$. In the contrast, few results are known if the boundary measurement is limited to a single pair $(f, \partial_\nu u)$. For such inverse inclusion problems, the geometry of D is a key point to proceed.

In this talk, I will present some recent results on this subject. In particular, I will focus on thee case when D is a convex polyhedron. I will share my thoughts on the corner singularity of the solution u and how to use it to determine D from a single boundary measurement.

Transmission problem for perfect conductor with small interface resistance

Shota Fukushima (Gunma University)

Abstract:
We consider a transmission problem for Laplace equation with interface resistance. Interface resistance appears as a controlling parameter for discontinuity of potentials across the interface. In application, it is naturally expected that the potential converges to that for ideal interface as the interface resistance tends to zero. In this talk, we prove this conjecture in a mathematically rigorous way. The key idea is to reduce the transmission problem to a functional equation on boundary involving layer potentials and Dirichlet-to-Neumann map associated with an exterior Dirichlet problem. This talk is based on joint work with Yong-Gwan Ji (KIAS) and Hyeonbae Kang (Inha University).

Neutral inclusions with imperfect bonding

Yong-Gwan Ji (Korea Institute for Advanced Study)

Abstract:
In this talk, we study neutral inclusions with imperfect bonding, namely inclusions that do not perturb the background potential. We consider two representative imperfect interface models, the LC-type and HC-type conditions, and investigate the shapes of inclusions and interface parameters for which neutrality holds for all uniform fields. In particular, we discuss classification results for neutral inclusions in the LC- and HC-type cases.

Coupling the coupled complex boundary method and ADMM for constrained shape optimization in geometric inverse problems

Julius Fergy Tiongson Rabago (Kanazawa University, Japan)

Abstract:
This talk addresses geometric inverse problems through a shape optimization framework, with applications to the identification of unknown cavities or obstacles from boundary measurements in non-destructive testing. The approach is based on the coupled complex boundary method (CCBM), which provides a complex-valued formulation for the reconstruction of the unknown interior boundary. To improve the detection of non-convex features, in particular concave regions, we introduce additional inequality constraints within the shape optimization problem. The resulting constrained problem is solved using the alternating direction method of multipliers (ADMM). The formulation requires particular care, as the CCBM cost functional depends on the imaginary part of the state variable, while the inequality constraints involve

its real part. A suitable splitting strategy is proposed to ensure a consistent integration of CCBM within the ADMM framework. The resulting CCBM–ADMM algorithm provides stable iterations and improved reconstruction properties. Numerical experiments in two and three dimensions, including noise-contaminated data, show that the proposed method outperforms standard first-order shape optimization approaches, especially in reconstructing geometries with pronounced concave features.

Exploring heuristic rules for iterative and variational regularization for inverse problems

Rommel R. Real (University of the Philippines Mindanao)

Abstract:
We consider both the Landweber iteration and variational regularization for solving linear as well as nonlinear inverse problems in Banach spaces. Based on the discrepancy principle, we proposed a heuristic parameter choice rule for choosing the regularization parameter which does not require the information on the noise level, so it is purely data-driven. By imposing certain conditions on the noisy data, we discuss a new convergence result which requires neither the Gâteaux differentiability of the forward operator nor the reflexivity of the image space. This expands to non-smooth ill-posed inverse problems and to situations that the data is contaminated by various types of noise. We also discuss some challenges along the way, along with numerical simulations. Future directions will also be discussed, pushing new trends on regularization.

Frequency dependent contraction rates for the Bayesian method to the inverse source problem

Pu-Zhao Kow (National Chengchi University)

Abstract:
We study an inverse source problem for acoustic waves over a range of frequencies from two complementary perspectives.

- First, although the problem is severely ill-posed (exhibiting only logarithmic stability), we show, through a detailed analysis of the singular values of the forward operator, that enlarging the frequency range enhances stability. This phenomenon is commonly referred to as increasing resolution/stability.
- Second, motivated by this singular value behavior, we establish a consistency result within a nonparametric Bayesian framework. Despite the ill-posedness, the posterior distribution contracts around the true source at a rate involving both polynomial and logarithmic terms, with explicit dependence on the frequency range. This captures the increasing resolution/stability phenomenon from a Bayesian viewpoint.

Our results also offer practical insights into how to balance the quantity, quality, and cost of measurements. This talk is based on joint work with Jenn-Nan Wang.

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Day 2

Inverse boundary problems with a single passive observation

Hongyu Liu (City University of Hong Kong)

Abstract:
This talk presents our recent progress in solving inverse boundary problems using only a single passive observation. The key to this result is a suite of new technical tools, chief among them a dimensional reduction technique (either parametric or geometric) and an operator-theoretic shift designed for nonlinear PDEs. These developments enable unique recovery of unknown parameters where previously multiple, active measurements were required.

An inverse problem for semilinear elliptic equations with generalized Kerr-type nonlinearities

Rulin Kuan (National Cheng Kung University)

Abstract:
We study the inverse problem of reconstructing the shape of unknown inclusions in semilinear elliptic equations with nonanalytic nonlinearities, by extending Ikehata's enclosure method to accommodate such nonlinear effects. To address the analytical challenges, we construct an approximate solution based on the linearized equation, enabling the enclosure method to operate in this setting. Under suitable structural conditions on the nonlinearity, we establish a shape reconstruction result. The proposed method applies to a broad class of semilinear elliptic equations with non-analytic nonlinearities, including representative examples such as the Kerr-type nonlinearity and the Ginzburg-Landau-type nonlinearity.

MUSIC Imaging with Rotating Mono- and Bistatic Configurations

Sangwoo Kang (Graduate School of Data Science, Pusan National University, Pusan 46241, Korea)

Abstract:
In this work, we investigate the performance and structural properties of the MUSIC (Multiple Signal Classification) imaging method under rotating monostatic and bistatic measurement configurations. We consider a setting in which the transmitter–receiver pair moves synchronously along the unit circle while maintaining a fixed bistatic angle. This configuration enables a unified treatment of mono- and bistatic data and leads to a structured dependence of the measurements on the measurement geometry.

Based on this framework, we analyze the resulting Multi-Static Response (MSR) matrix and identify its underlying structure, which can be described through analytical expressions involving Bessel functions. This characterization provides insight into the resolution and localization behavior of the MUSIC imaging functional. In particular, we examine how the imaging performance depends on the bistatic angle and the rotation scheme, and we clarify the conditions under which the classical monostatic behavior is recovered as a limiting case.

Numerical experiments are presented to support the theoretical analysis and to demonstrate the effectiveness of the proposed configuration in practical imaging scenarios. The results indicate that the proposed framework provides a unified perspective for analyzing and improving MUSIC-based imaging in both monostatic and bistatic settings.

Latent Autoencoder Ensemble Kalman Filter for Data assimilation

Tong Xin (National University of Singapore)

Abstract:
The ensemble Kalman filter (EnKF) is widely used for data assimilation in high-dimensional

systems, but its performance often deteriorates for strongly nonlinear dynamics due to the structural mismatch between the Kalman update and the underlying system behavior. In this work, we propose a latent autoencoder ensemble Kalman filter (LAE-EnKF) that addresses this limitation by reformulating the assimilation problem in a learned latent space with linear and stable dynamics. The proposed method learns a nonlinear encoder–decoder together with a stable linear latent evolution operator and a consistent latent observation mapping, yielding a closed linear state-space model in the latent coordinates. This construction restores compatibility with the Kalman filtering framework and allows both forecast and analysis steps to be carried out entirely in the latent space.

Inverse Fiedler vector problem of a graph

Jephian C.-H. Lin (National Yang Ming Chiao Tung University)

Abstract:
Given a graph and one of its weighted Laplacian matrix, a Fiedler vector is an eigenvector with respect to the second smallest eigenvalue. The Fiedler vectors have been used widely for graph partitioning, graph drawing, spectral clustering, and finding the characteristic set. This paper studies how the graph structure can control the possible Fiedler vectors for different weighted Laplacian matrices. For a given tree, we characterize all possible Fiedler vectors among its weighted Laplacian matrix. As an application, the characteristic set can be anywhere on a tree, except for the set containing a single leaf. For a given cycle, we characterize all possible eigenvectors corresponding to the second or the third smallest eigenvalue.

This is a joint work with Mahsa N Shirazi.

Transformers through the lens of support-preserving maps between measures

Takashi Furuya (Doshisha University)

Abstract:
Transformers are deep architectures that define "in-context maps" which enable predicting new tokens based on a given set of tokens (such as a prompt in NLP applications or a set of patches for a vision transformer). Previous work has studied the ability of these architectures to handle an arbitrarily large number of context tokens. To mathematically, uniformly analyze their expressivity, previous work considered the case that the mappings are conditioned on a context represented by a probability distribution which becomes discrete for a finite number of tokens. Modeling neural networks as maps on probability measures has multiple applications, such as studying Wasserstein regularity, proving generalization bounds and doing a mean-field limit analysis of the dynamics of interacting particles as they go through the network. In this work, we study the question what kind of maps between measures are transformers. We fully characterize the properties of maps between measures that enable these to be represented in terms of in-context maps via a push forward. On the one hand, these include transformers; on the other hand, transformers universally approximate representations with any continuous in-context map. These properties are preserving the cardinality of support and that the regular part of their Fréchet derivative is uniformly continuous. Moreover, we show that the solution map of the Vlasov equation, which is of nonlocal transport type, for interacting particle systems in the mean-field regime for the Cauchy problem satisfies the conditions on the one hand and, hence, can be approximated by a transformer; on the other hand, we prove that the measure-theoretic self-attention has the properties that ensure that the infinite depth, mean-field measure-theoretic transformer can be identified with a Vlasov flow.

Mamba-Based Neural Operators for PDEs: Toward Efficient Forward and Inverse Problem Solving

Junseung Ryu (KAIST)

Abstract:
Neural operators have emerged as a powerful framework for learning solution maps of partial differential equations (PDEs), offering a data-driven alternative to classical numerical solvers. Recent works have largely relied on Transformer-based architectures; however, their quadratic complexity poses limitations in large-scale or real-time settings.

In this work, we propose a Mamba-based neural operator (MBNO), which replaces the self-attention mechanism with structured state space models (SSMs). We show both theoretically and empirically that Mamba layers can effectively approximate operator mappings while achieving improved accuracy and significantly faster inference compared to Transformer-based counterparts.

While our primary focus is on forward operator learning, we argue that such improvements are directly relevant to inverse problems. In this context, a fast and expressive forward neural operator serves as a critical computational backbone.

We support this perspective through experiments, including inverse Darcy flow, where the proposed Mamba-based framework demonstrates promising performance, particularly in handling discontinuous coefficients. These results suggest that Mamba-based neural operators provide a scalable and flexible foundation not only for forward simulations but also for inverse problem settings.

Finally, we discuss future directions toward fully integrating Mamba architectures into inverse operator learning, including extensions to cross-attention mechanisms and more general inverse formulations.

On the contraction rate of the posterior distribution for nonlinear PDE parameter identification

Yuxin Fan (The Chinese University of Hong Kong)

Abstract:
This talk concerns Bayesian estimation of unknown parameters in nonlinear inverse problems for PDEs, focusing on posterior distributions induced by Gaussian process priors and their variational approximations. We establish contraction rates for both the posterior distribution and its variational approximation in the low-regularity regime. A key contribution is the removal of the standard assumption that the true parameter belongs to the reproducing kernel Hilbert space (RKHS) of the prior, which substantially broadens the scope of existing posterior contraction results. Our analysis is based on constructing a sequence of RKHS approximations to the low-regularity true parameter and characterizing the trade-off between the approximation error and the growth of the associated RKHS norms. The resulting theory is illustrated through three nonlinear PDE inverse problems.